Fall 2021 (110-1)

控制系統
Control Systems

Unit 50 & Unit 60
Specifications & Requirements for Root Locus & Bode Plot

Feng-Li Lian

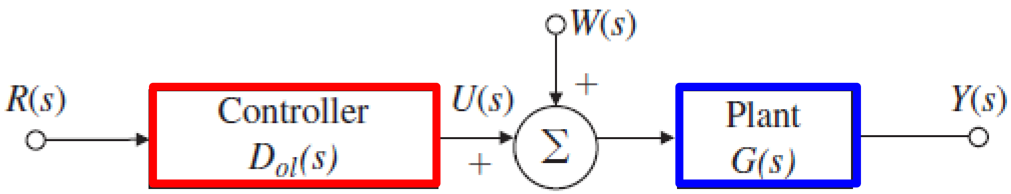
NTU-EE

Sep 2021 – Jan 2022

Open-Loop and Closed-Loop Systems

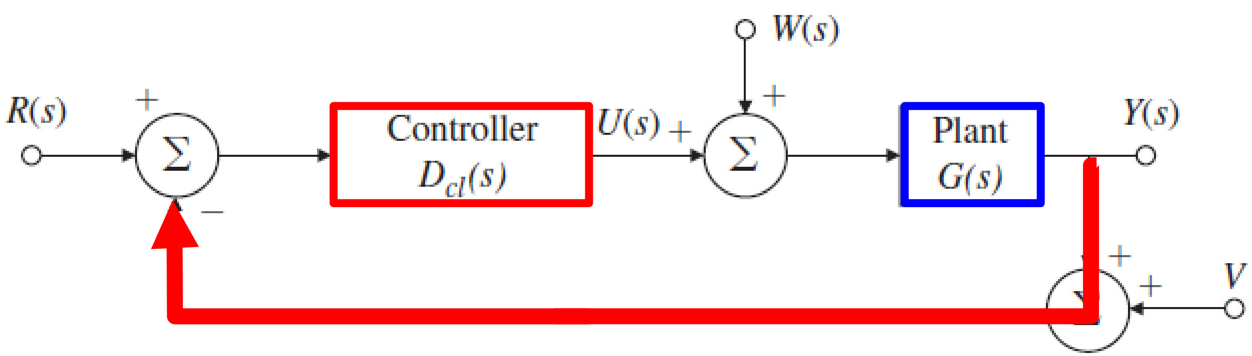
- Open-loop system showing
  reference, R, control, U, disturbance, W, and output Y

$$G = \frac{b(s)}{a(s)} \qquad D = \frac{c(s)}{d(s)}$$



$$T_{ol} = \frac{Y_{ol}}{R} = G\, D_{ol} = \frac{b(s)}{a(s)}\,\frac{c(s)}{d(s)}$$

$$T_{cl} = \frac{Y_{cl}}{R} = \frac{G\, D_{cl}}{1 + G\, D_{cl}} = \frac{b(s)\, c(s)}{1 + a(s)\, d(s)}$$

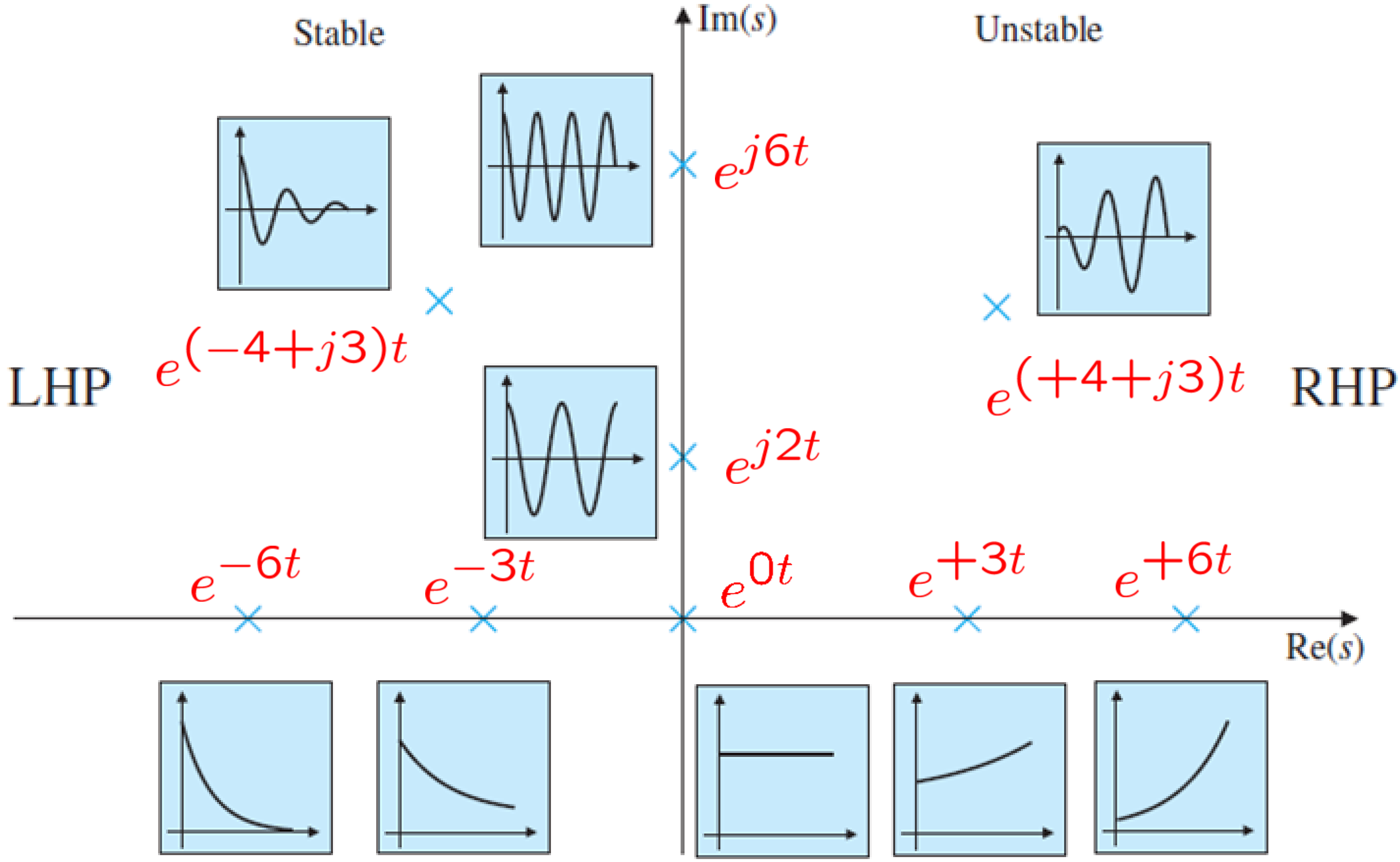


- Poles
  of Transfer Function
- Roots
  of Characteristic Equation

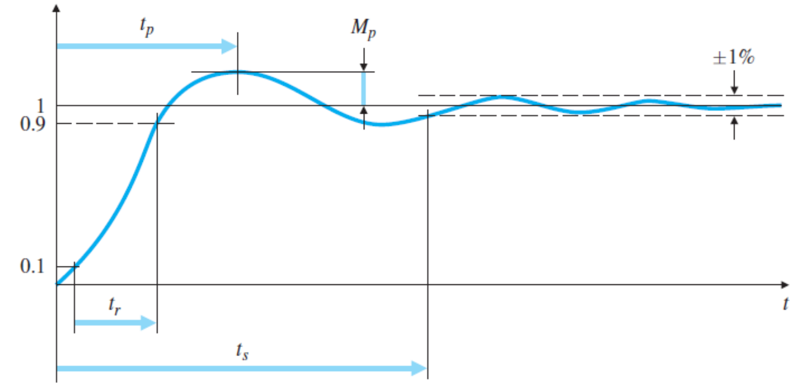
- Closed-loop system showing
  reference, R, control, U, disturbance, W, output, Y, and sensor noise, V

Effect of Pole Locations

- Time functions associated with points in the s-plane
  (LHP, left half-plane; RHP, right half-plane)



- Overshoot M_p and Peak time t_p



$$Y(s) = H(s)\,\frac{1}{s}$$

$$H(s) = \frac{{w_n}^2}{s^2 + 2\,\zeta\,w_n\,s + {w_n}^2}$$

$$\sigma = w_n\,\zeta$$

$$w_d = w_n\sqrt{1-\zeta^2}$$

$$\Rightarrow\ y(t) = 1 - e^{-\sigma t}\left(\cos w_d t + \frac{\sigma}{w_d}\sin w_d t\right)$$

$$\Rightarrow\ y(t) = 1 - \frac{e^{-\sigma t}}{\sqrt{1-\zeta^2}}\cos(w_d t - \beta)$$

$$A\sin(\alpha) + B\cos(\beta) = C\cos(\alpha - \beta)$$

$$C = \sqrt{A^2 + B^2} = \frac{1}{\sqrt{1-\zeta^2}}$$

$$\beta = \tan^{-1}\left(\frac{A}{B}\right)$$

$$= \tan^{-1}\left(\frac{\zeta}{\sqrt{1-\zeta^2}}\right) = \sin^{-1}(\zeta)$$

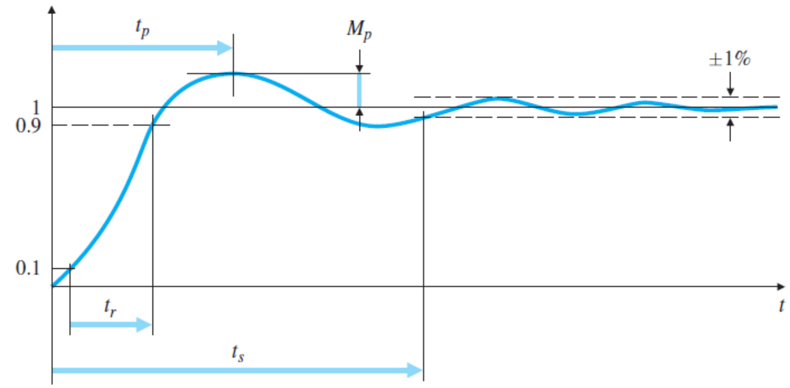
$$\Rightarrow\ M_p = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$$

$$\Rightarrow\ t_r \cong \frac{1.8}{w_n}$$

$$\Rightarrow\ t_s = \frac{4.6}{\zeta w_n} = \frac{4.6}{\sigma}$$

U3D Time-Domain Specifications

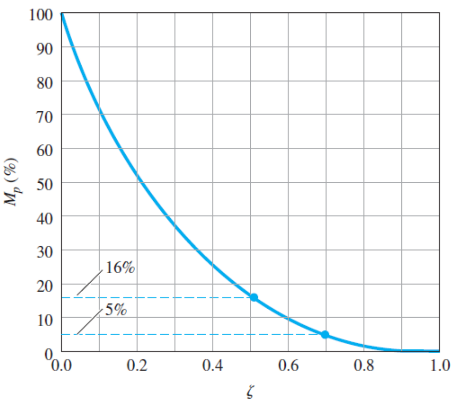
- Rise time t_r
- Settling time t_s
- Overshoot M_p
- Peak time t_p

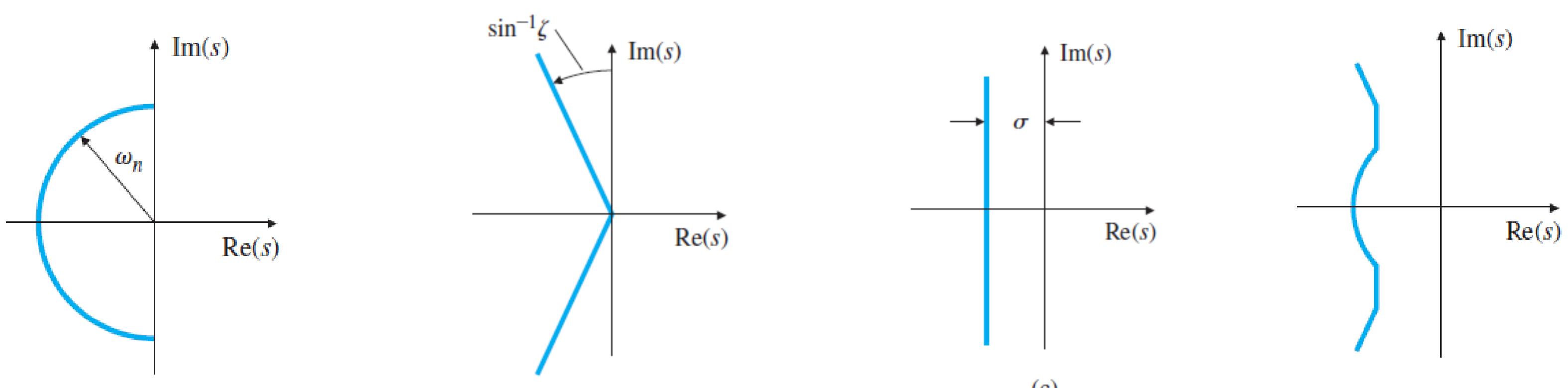


$$\Rightarrow t_r \cong \frac{1.8}{w_n} \qquad \Rightarrow w_n \geq \frac{1.8}{t_r}$$

$$\Rightarrow t_s = \frac{4.6}{\zeta w_n} = \frac{4.6}{\sigma} \qquad \Rightarrow \zeta \geq \zeta(M_p)$$

$$\Rightarrow M_p = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} \qquad 0 \leq \zeta < 1 \qquad \Rightarrow \sigma \geq \frac{4.6}{t_s}$$



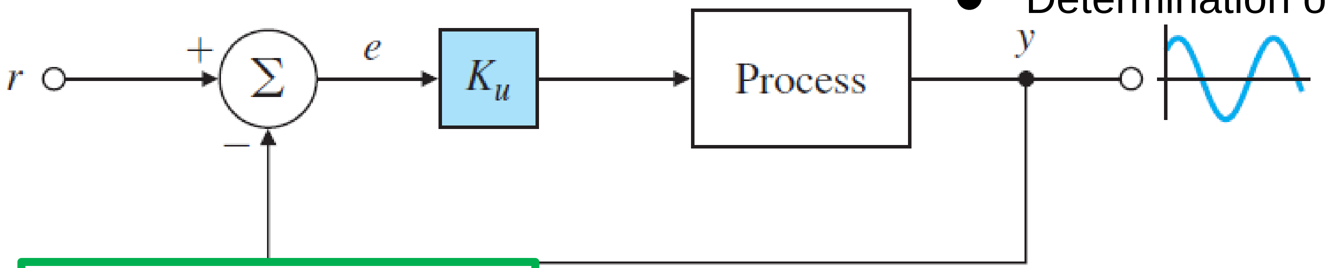


(a) (b) (c) (d)

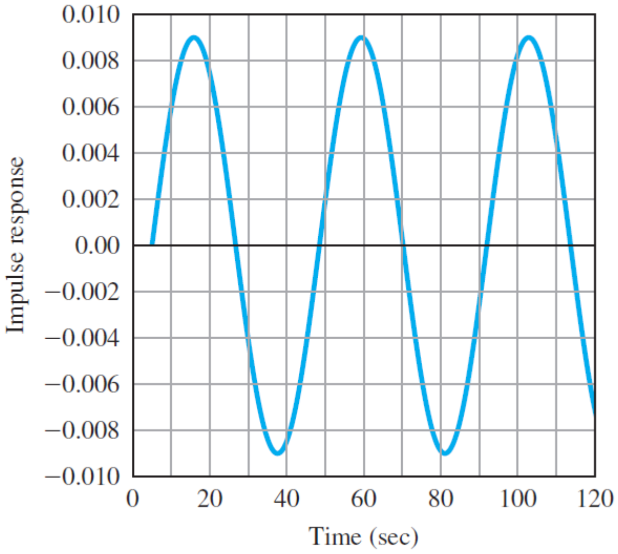
- **Method 2: Ultimate Sensitivity Method:**

  Based on evaluating the amplitude and frequency of the oscillations of the system at the limit of stability rather than on taking a step response.

  - Determination of ultimate gain and period



- K_u : Ultimate Gain
- P_u : Ultimate Period



Ziegler-Nichols Tuning for the Regulator $D_c(s) = k_P(1 + 1/T_I s + T_D s)$, Based on the Ultimate Sensitivity Method

Type of Controller	Optimum Gain
P	$k_P = 0.5K_u$
PI	$k_P = 0.45K_u$, $T_I = \frac{P_u}{1.2}$
PID	$k_P = 0.6K_u$, $T_I = 0.5P_u$, $T_D = 0.125P_u$

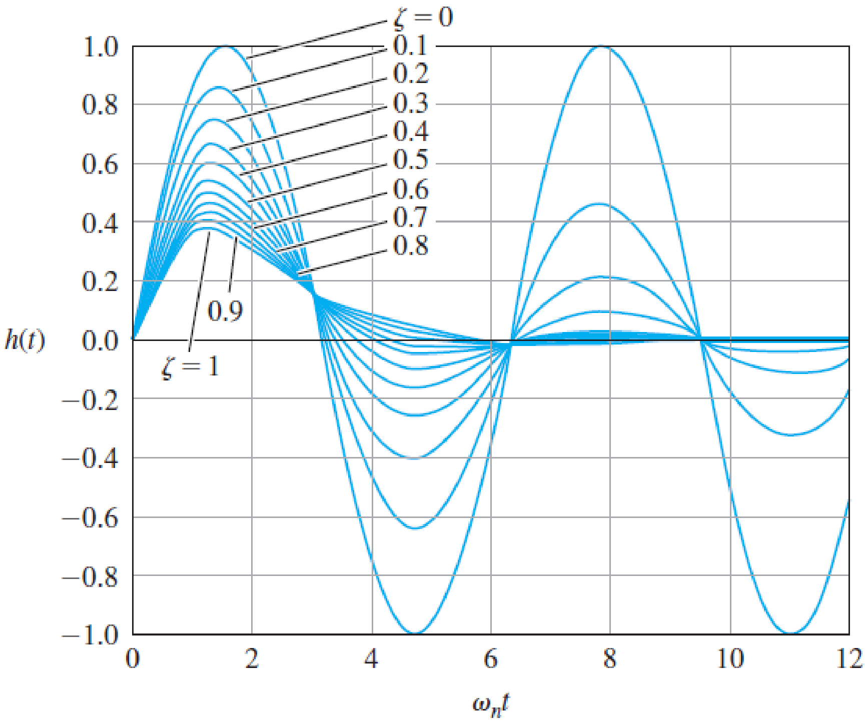
Complex Poles: Damping Ratio, Undamped Natural Frequency

$$H(s) = \frac{{w_n}^2}{(s+\zeta w_n)^2 + {w_n}^2(1-\zeta^2)} \qquad \sigma = w_n \zeta$$

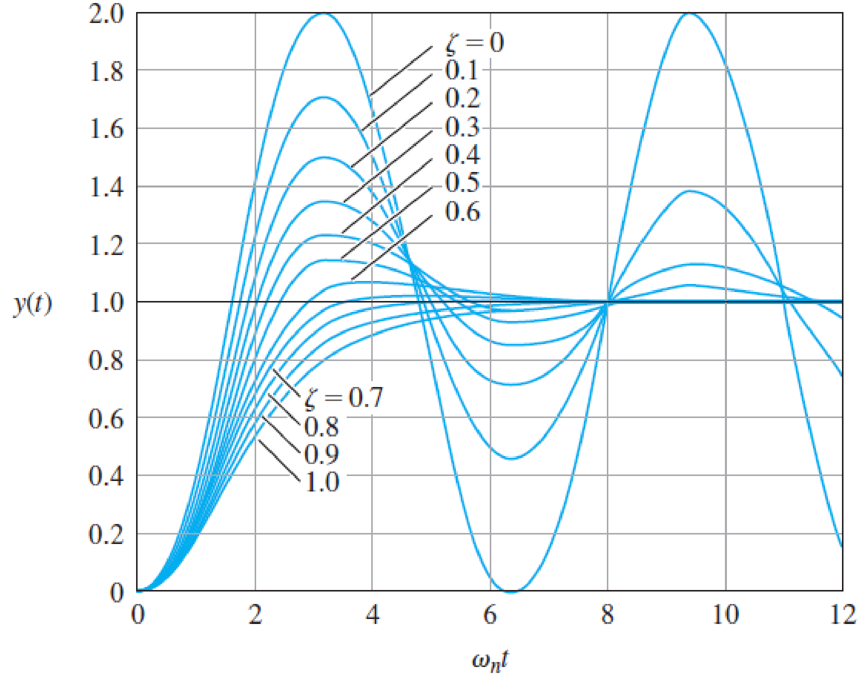
$$h(t) = \frac{w_n}{\sqrt{1-\zeta^2}} e^{-\sigma t} (\sin w_d t)\, 1(t) \qquad w_d = w_n \sqrt{1-\zeta^2}$$

- Responses of second-order systems versus ζ :

(a) Impulse Responses (b) Step Responses



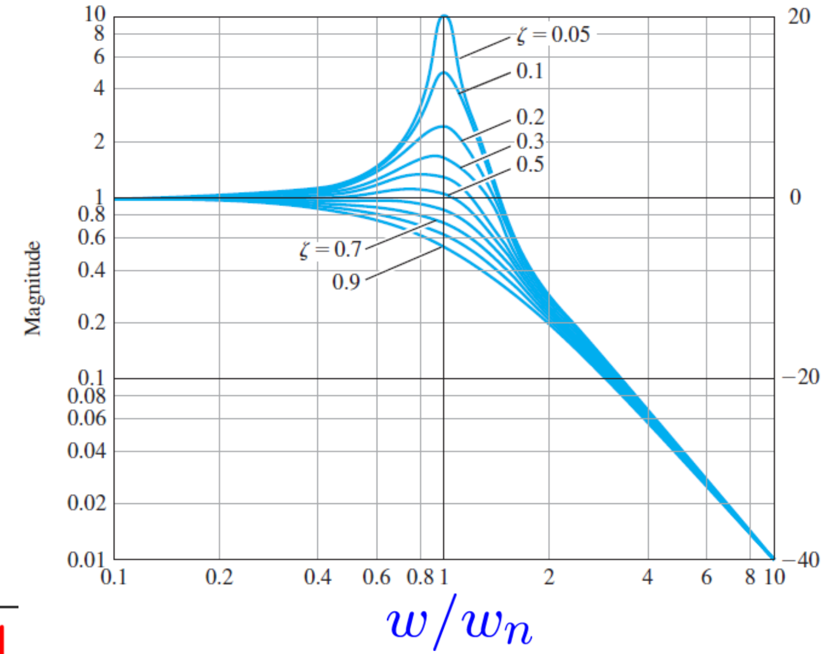
(a)



(b)

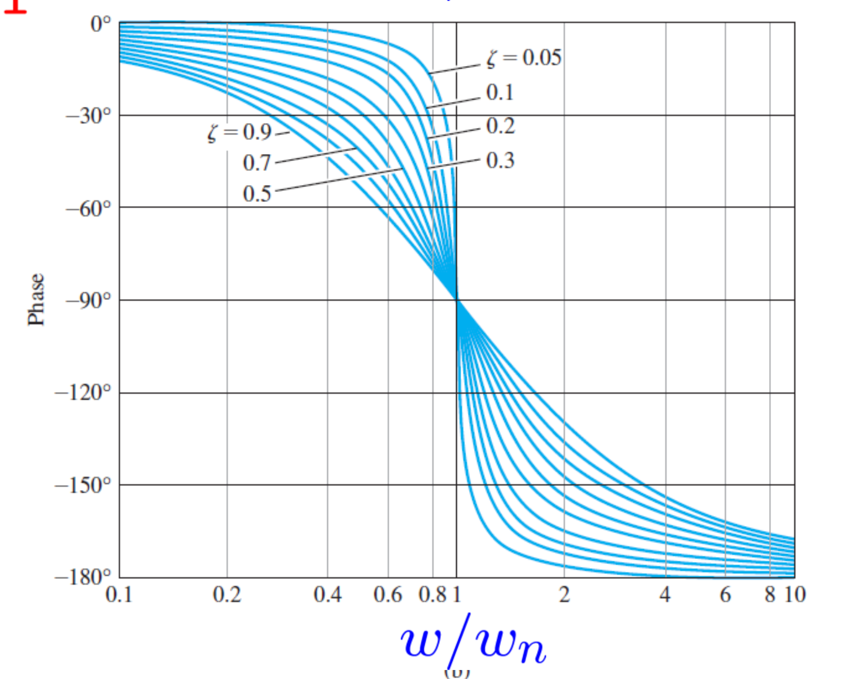
Frequency Response and Bode Plot

$$|G(s = jw)|$$



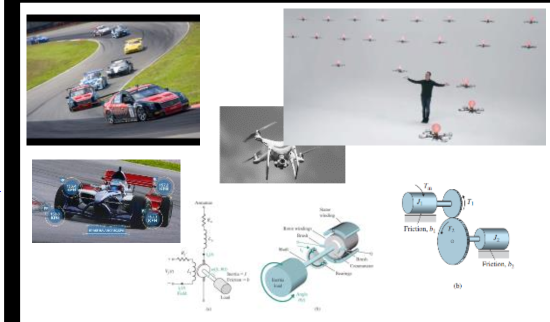
$$G(s) = \frac{1}{(s/w_n)^2 + 2\zeta(s/w_n) + 1}$$

$$\measuredangle G(s = jw)$$



Plant, Input, Output, Action, Goal

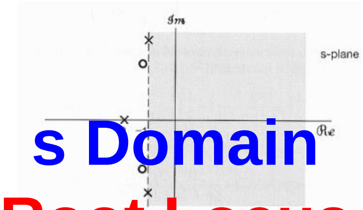
Plant (P)

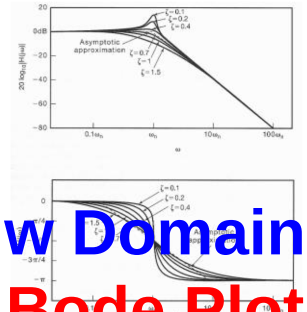


Ref (r) **Input (u)** **Output (y)**

$$\frac{d^2y(t)}{dt^2} + 2\,\frac{dy(t)}{dt} - 3\,y(t) = 5\,u(t)$$

$$P(s) = \frac{Y(s)}{U(s)} = \frac{5}{s^2 + 2\,s - 3}$$





s Domain
Root Locus

w Domain
Bode Plot

Controller

1. **Model**
2. **Response**
3. **Analysis**
4. **Feedback**
5. **Control**

$$\frac{d^2y(t)}{dt^2} + 4\,\frac{dy(t)}{dt} + 3\,y(t) = 3\,r(t)$$

$$G(s) = \frac{Y(s)}{R(s)} = \frac{3}{s^2 + 4\,s + 3}$$